Chapter 14
Gauss's Law Chapter Review

EQUATIONS:

- $\Phi_E = \int_S \mathbf{E} \bullet d\mathbf{S}$ [The total electric flux Φ_E through a surface S is determined by defining a differential surface area vector dS (this is a vector whose magnitude is the area of the differential surface and whose direction is ALWAYS perpendicularly outward from the surface), determining the differential flux through that differential surface by dotting dS into the electric field vector evaluated at dS, then integrating to sum up all of the differential flux quantities over the entire surface.]

- $\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{enclosed}}{\varepsilon_o}$ [Gauss's Law states that the net electric flux through a closed surface will be proportional to the charge enclosed within the surface. The constant $\frac{1}{\varepsilon_o}$ allows the expression to become an equality. Gauss's Law is ALWAYS TRUE, but it's only useful if you can find an imaginary Gaussian surface--a sphere or cylinder in most cases--upon which the electric field is either a constant or is such that E·dS = 0.]

- $q_{within\ a\ volume} = \int dq = \int \rho dV$ [The amount of charge q in a structure is equal to the sum of the differential charges dq within all of the differential volumes making up the structure. The differential charge dq within a differential volume dV will equal the product of the charge per unit volume ρ EVALUATED AT THE POINT OF INTEREST and dV, or dq = ρ dV. Additionally, the trick is to write the differential volume dV in terms of variables associated with the geometry of the structure (i.e., in terms of r's and/or x's), and to select appropriate limits of integration (i.e., to integrate from where the charge starts to the edge of a Gaussian surface, or from where the charge starts to where the charge ends, depending upon the situation).]

- $dV_{sphere} = (4\pi r^2)dr$ [This is the differential volume of a spherical shell of radius r and thickness dr.]

- $dV_{cylinder} = (2\pi rL)dr$ [This is the differential volume of a cylindrical shell of radius r, length L, and thickness dr.]

- $q_{on\ a\ surface} = \int dq = \int \sigma dS = \sigma A$ [Although the $\int \sigma dS$ representation is technically correct, you will NEVER have to deal with that integral because you will NEVER run into a surface charge density function σ that is variable. Why? Because such a function would produce a situation in which the symmetry required for Gauss's Law would not exist. That means that the charge on any surface that has a surface charge density function associated with it will always equal σ (it will just be a number--no variability) times the surface area, or $q_{on\ the\ surface} = \sigma A$.]

- $q_{on\ L's\ worth\ of\ wire} = \lambda L$ [The amount of charge on a length of wire L equals the charge per unit length λ times L.]

- $E = \dfrac{\sigma}{2\varepsilon_o}$ [This is the expression for the electric field VERY CLOSE to the central surface of a charged INSULATOR. Note that the charge per unit area function in this case defines the amount of charge that is shot through the volume BEHIND a unit area of the surface.]

- $E = \dfrac{\sigma}{\varepsilon_o}$ [This is the expression for the electric field VERY CLOSE to the central surface of a charged CONDUCTOR. Note that the charge per unit area function in this case defines the amount of charge that exists in a unit area ON ONE SIDE OF THE PLATE.]

<u>COMMENTS, HINTS, and THINGS to be aware of</u>:

- All Gauss's Law says is that the charge inside a closed surface is proportional to the electric flux through that surface.

- When doing the right side of Gauss's Law (i.e., the charge enclosed part), be sure to include the sign. If, upon solving for the magnitude of E, you end up with a negative quantity, the negative sign simply means the field is opposite to the direction you assumed. (For the sake of simplicity, I always start by assuming that E is outward from the closed surface. See the next note.)

- If the charge configuration producing an electric field has spherical symmetry, the left side of Gauss's Law will ALWAYS be $\int_S \mathbf{E} \bullet d\mathbf{S} = \int_S (E)(dS)\cos 0^o = E\int_S dS = E(4\pi r^2)$, where r is the radius of the spherical Gaussian surface, $4\pi r^2$ is the area of that surface, and the surface has been placed symmetrically about the charge. Note that E is assumed to be oriented in the same direction as dS, which is to say, outward from the surface, so that the angle between the two vectors is zero and the cosine associated with the dot product is one. This is arbitrary, but within the context of Gauss's Law, it works out nicely. That is, if the charge happens to be negative, suggesting that E should be inward, Gauss's Law produces an electric field expression that is negative. The negative sign simply means you have assumed the wrong direction for E and E is really inward.

- If the charge configuration producing an electric field has cylindrical symmetry, the left side of Gauss's Law will ALWAYS be $\int_S \mathbf{E} \bullet d\mathbf{S} = \int_S (E)(dS)\cos 0^o = E\int_S dS = E(2\pi r L)$, where r is the radius, L is the length of the cylindrical Gaussian surface, $2\pi rL$ is the area of the cylindrical part of the surface, and the imaginary Gaussian cylinder has been placed symmetrically about the charge. Again, E is assumed to be directed outward from the cylindrical part of the surface. This means there is no electric flux coming out of the cylinder's ends because dS is perpendicular to E over that part of the structure.

- Be careful not to forget any charge enclosed within your Gaussian surface. In situations in which, say, a volume charge density function is defined, people often get so excited about mastering the integral $\int \rho dV$ that they completely forget any other charge that might exist within the surface.

- Inside a CONDUCTOR, E MUST be zero if we are talking about a static electric field. How so? If the field is non-zero, charge will move in the structure until there is no more net force acting on it. In other words, charge will rearrange itself until the net electric field is zero.

  --A consequence of this? The charge $q_{enclosed}$ inside a Gaussian surface located within a conductor MUST BE ZERO. If that means that charge within the structure has to redistribute itself, polarizing on the inside and outside surfaces, so be it. One way or the other, the net charge inside a Gaussian surface located within a conductor must sum to zero.